**BOOK 2****NEW MATHEMATICS****Practice Workbook****SETS AND SUBTRACTION**

Exercises which develop the basic ideas and terms used in New Mathematics programs, with easy-to-understand explanatory notes for parents and teachers.



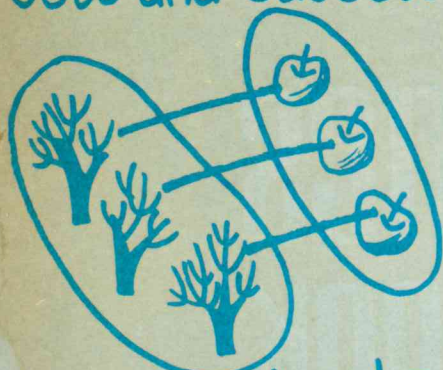
sets and subsets

equalities
 $5 + \square = 7$
 $5 < 6$
inequalities



sets and subsets

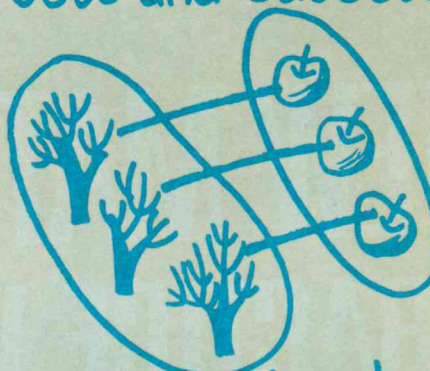
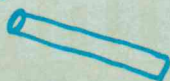
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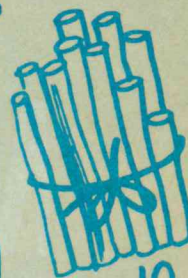
equivalent sets



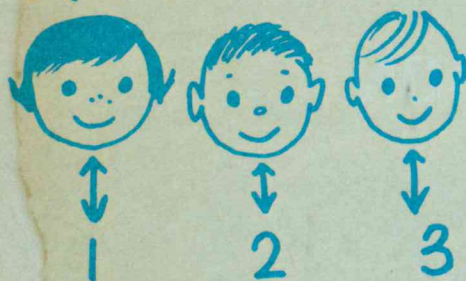
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and 1 more



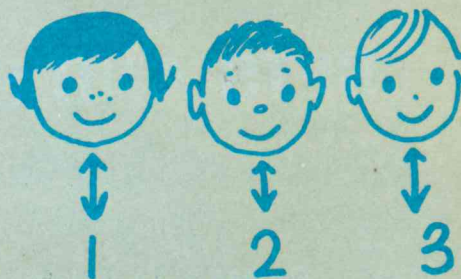
equivalent sets



10
and 1 more



one-to-one
matching



one-to-one
matching



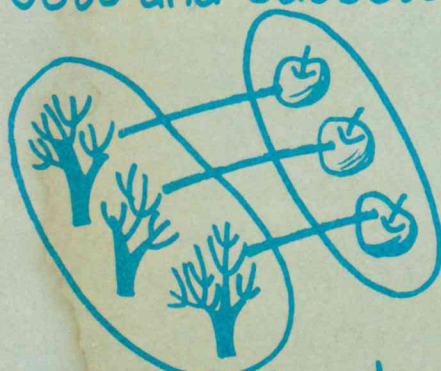
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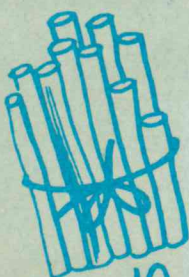


sets and subsets

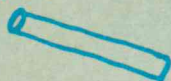
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equivalent sets



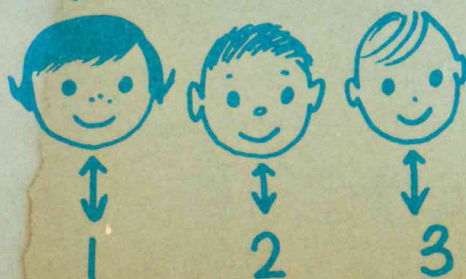
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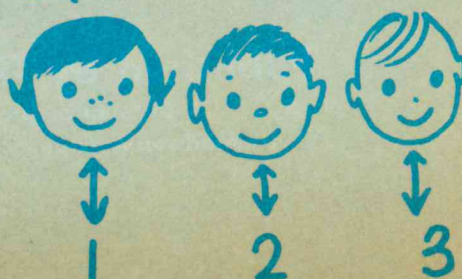
equivalent sets



10
and 1 more



one-to-one
matching



one-to-one
matching

BOOK 2

NEW MATHEMATICS PRACTICE WORKBOOK

SETS AND SUBTRACTION

By MARCIA LEVIN and JEANNE BENDICK

Illustrated by JEANNE BENDICK

Authors of "Take a Number,"
"Take Shapes, Lines and Letters"
and "Mathematics Illustrated Dictionary."



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Note to Parents and Teachers

What is the new mathematics?

Why is it being used in so many school systems today?

Can you help your children with the new mathematics?

You *can* help your children. You will find in this book an easy step-by-step development of the concepts and terminology of the new mathematics.

Teachers and educators today do not assume that most children will become professional mathematicians. What they do stress is that children will learn the underlying structure and pattern of all mathematics; that they will be able to move with ease from one area of mathematics to another; that they will be able to understand what they are doing and why, instead of memorizing isolated facts and tables; and perhaps most important, that they will find mathematics an exciting and rewarding subject. True, they still learn the fundamentals — how to add, subtract, multiply and divide. But their learning now is geared to the “knowledge explosion” of our times.

Words in this book, such as *commutative property*, *empty set*, *inverse*, may seem strange at first, but as the practice exercises are developed it will be seen that the precision of language, and the careful building of one idea on another, make the “new” mathematics much clearer and more easily understood than the old.

It is important that the procedures given here be followed exactly. The notes at the bottom of each page serve as guides for additional practice, and help the adult amplify the work where necessary.

Although this book can be used independently, it would be useful to start with Book I, which introduces and explains more fully the work which is reviewed on pages 3-22.

Working in this book will probably be as stimulating an experience for the adult who guides the child as for the child himself, and the “new” mathematics will reveal new horizons for adult and child alike.

The authors wish to thank W. Erwin Marriot, editor of many mathematics textbooks for the elementary schools, for his careful review of this book.

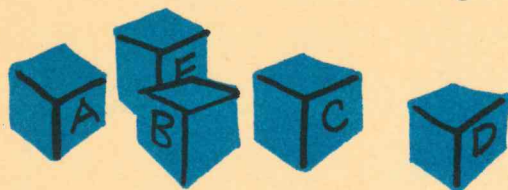
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SETS

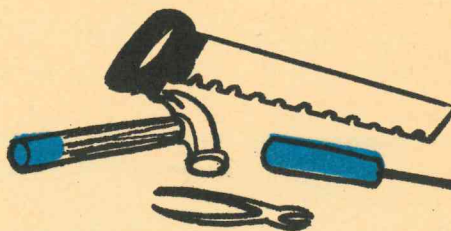
ELEMENTS

A set is a *group* of things or
a *collection* of things.



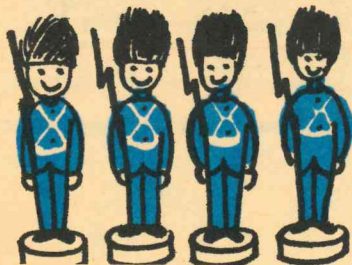
This is a set of blocks.

The things that belong to a set are called
the *members* of the set or
the *elements* of the set or
the *objects* in a set.

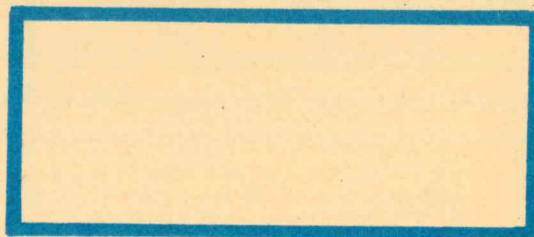


This is a set of tools.

The elements in a set can be all the same.



This is a set of toy soldiers.

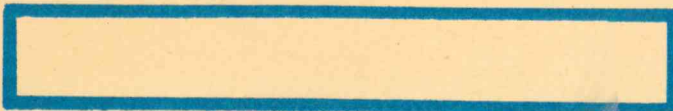
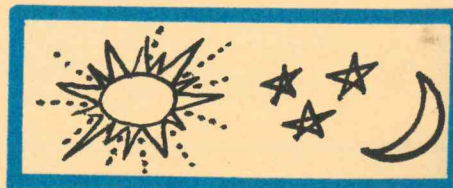


Draw a set of cups.

The elements do not have to be the same.

This is a set of things in the sky.

Draw a set of things that could
be on a table.



NOTE: The elements in a set must have a property in common, even if that property is just the fact that they are grouped together. The fruit in a basket, the things in your pocket, the pets in your neighborhood, may all be considered sets.

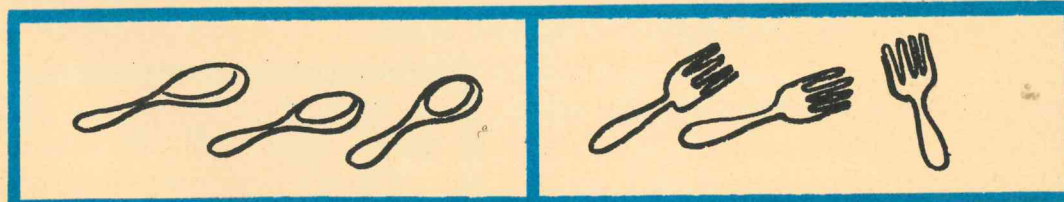
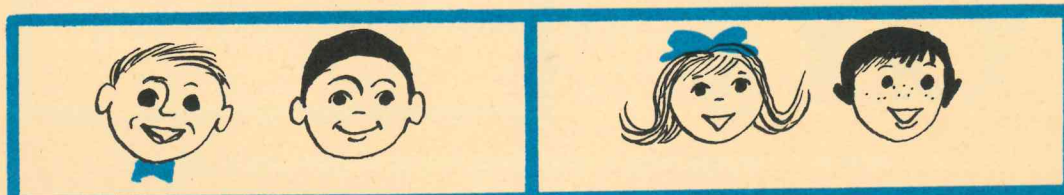
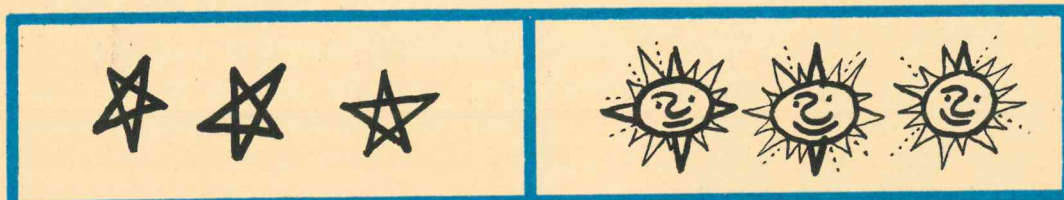
ONE-TO-ONE MATCHING EQUIVALENT SETS

A line can be drawn between each member of the first set and each member of another set.



This is called *one-to-one matching* or *one-to-one correspondence*.

Match the members of these sets one-to-one.



If the sets that are matched have the same number of elements, they are called *equivalent sets*.

NOTE: Each pair of sets on this page can be matched one-to-one, by drawing lines between the elements. Therefore these are all equivalent sets.

ONE-TO-ONE MATCHING

EQUIVALENT AND NON-EQUIVALENT SETS

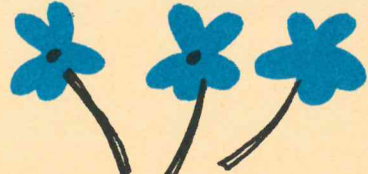
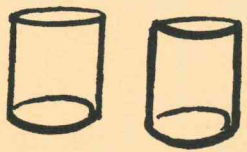
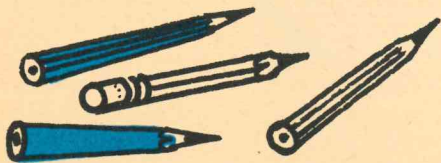
When members of sets can be matched one-to-one, with no members left over, the sets are called *equivalent sets*.

When members of sets cannot be matched one-to-one the sets are called *non-equivalent sets*.

Match the members of the sets below one-to-one.

If they are equivalent, write an E next to them.

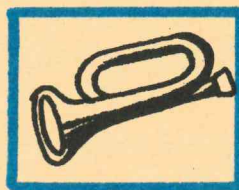
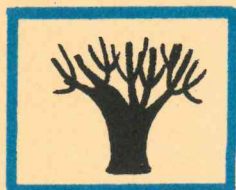
If they are non-equivalent, write an N next to them.

		☐
		☐
		☐

NOTE: One-to-one matching is a way of finding out "more than," "less than," or "as many as," without using numerals. Sheep were matched with pebbles, tallies were kept with marks or notches, before numerals were used. Even today the score of a game is sometimes kept by matching, or tallying, like this **||||**, instead of counting.

NUMBERS AND NUMERALS

What is the word that tells "how many" in each set below?



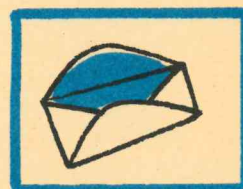
The name we give to the number of things in each of the sets above is *one*.

If everybody agreed to call that number of things "ab," then "ab" would be the name for that number of things.

We are used to the name *one*.

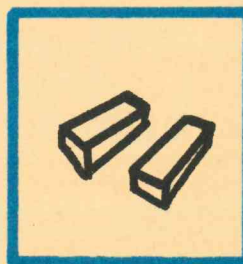
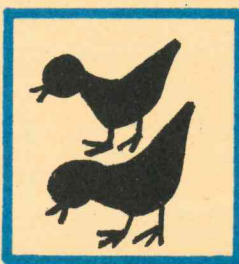
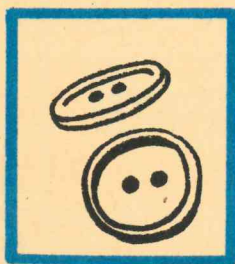
One is the name for this number of things:

We write *one* like this: 1



The symbol for a number is called a *numeral*.

What is the name we give to the number of things in each set below?



We call that number of things *two*.

We write *two* like this: 2

2

The word which we give to a certain number of things is called its number name.

The symbol for a number is a numeral.

NOTE: Have the child point out, or draw, sets containing 1 or 2 elements. Have him write the numeral for each set. *Numeral* is used when referring to the symbol representing a certain number of things.

NUMBERS TO FOUR

How many squares are in the strip below?

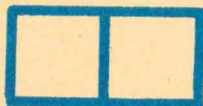


Write the numeral 1 in the first square.

Write the numeral 2 in the second square.

Here is a strip just like the strip above.

Write the numerals in the squares as you did above.



Draw another strip just like it, below the strip already drawn.

Write in the numerals as you did above.

Now draw one more square next to the square with the 2.

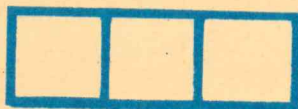
There are now *three* squares.

The numeral for three is written 3.

Write 3 in the new square above.

Here is a strip with three squares.

Write the numerals 1, 2, and 3 in the squares.



Draw another strip with 3 squares, just below the one already there. Write the numerals as you did above.

Draw one more square next to the square with the numeral 3.

There are now *four* squares.

The numeral is written 4.

Write 4 in the new square.

NOTE: Point out that 4 in type is slightly different from the handwritten 4.

NUMBERS TO FIVE

THE SET OF COUNTING NUMBERS

Starting with the square at the left in the strip below, write the numerals 1, 2, 3, and 4.



Draw another strip with 4 squares, below the one already drawn. Write in the numerals as you did above.

Draw one more square in the new strip next to the square with the numeral 4.

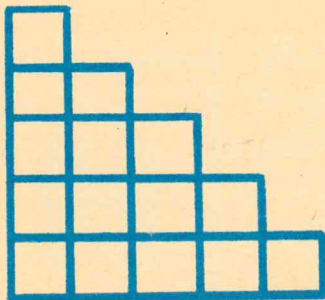
Now there are *five* squares.

The numeral for five is written 5.

Write 5 in the new square.

Write the numerals in the strips below.

Start at the left of each row with 1.



Each strip has one more square than the one above it.

Each strip has one more numeral than the one above it.

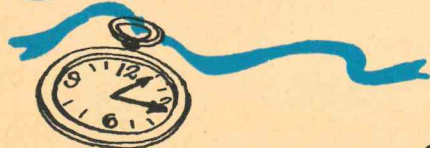
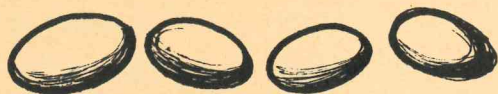
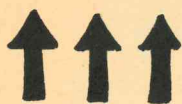
The last strip shows the first five numerals in the set of counting numbers.

Write the numerals for the first five counting numbers.

NOTE: The child should realize that the 2-strip is a set of 2 squares, the 3-strip is a set of 3 squares, etc. If additional practice is needed, the child should name, or draw, sets containing from 1 to 5 elements. He should be able to tell the number of elements in each set, and to write the numeral for that set.

SETS TO FIVE

Draw a line from each set to the numeral which stands for the number of elements in the set.



1
2
3
4
5
1
2
3
4
5

NOTE: The child should be able to recognize sets containing up to 5 elements without counting. He should realize that the numerals are symbols which tell the number of objects in a set.

CARDINAL NUMBER OF A SET

A set of objects can be matched one-to-one with the set of counting numbers.



1

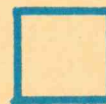
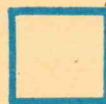
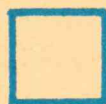
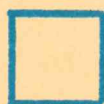
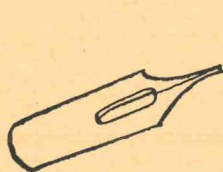
2

3

4

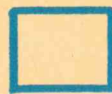
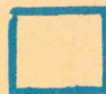
5

The box below each element in the set is called a *number frame*. Match the set of counting numbers with the elements in the set below by writing the numerals in the number frames.



The last number you name in counting the elements of a set is the *cardinal number* of the set.

Match the set of counting numbers with the set of cupcakes.



The cardinal number of the set is 5.

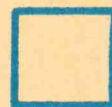
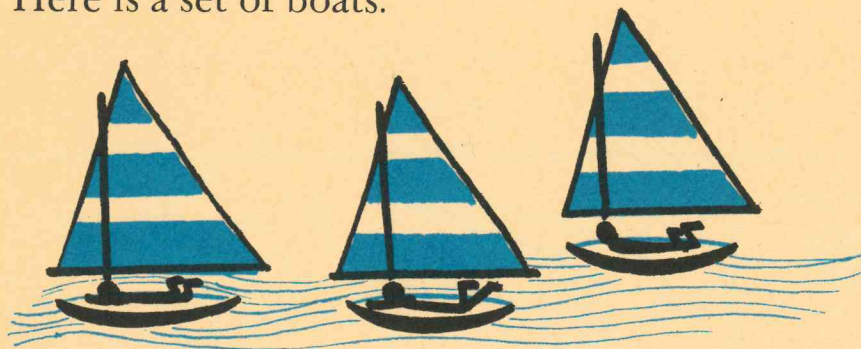
5

NOTE: Have the child practice matching the set of counting numbers with sets containing up to 5 elements. The last numeral is the cardinal number of the set.

SUBSETS PLUS

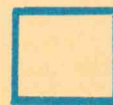
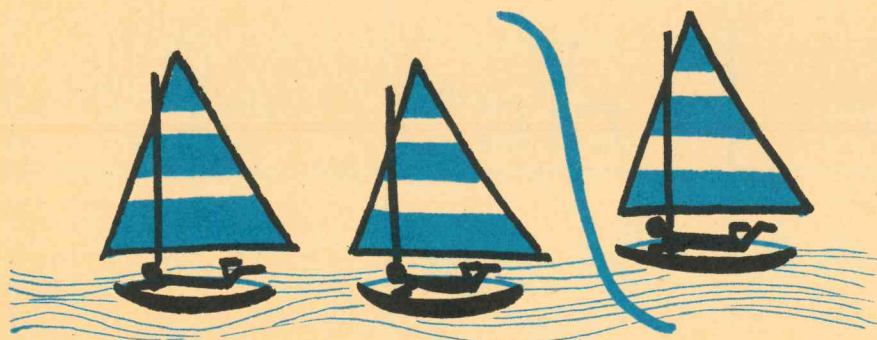


Here is a set of boats.



Write the cardinal number of the set.

We can draw a line separating the set of boats into two parts.



Now how many boats are in the set?

Drawing the line did not change the number of elements in the set.

Write the numerals to show how many boats are in each part.



Each part is called a *subset* of the set of 3.

2 and 1 more can be read

2 plus 1.

The plus sign is written $+$.

$2 + 1$ names the same number as 3.



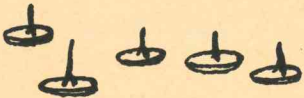
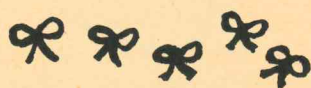
NOTE: Always read $+$ as *plus*, never *and*.
3, and $2 + 1$, name the same number.

RENAMING NUMBERS

SUBSETS

In the black number frame write the cardinal number of the set.

Then separate each set into any two subsets, and write the numerals that name the two subsets.

 +	 +	 +
 +	 +	 +
 +	 +	 +

When we separate a set into subsets, we are *renaming* the number of the set.

NOTE: Except for a set containing only one element, there is more than one way to write the number of elements in the set.

5 may be renamed:

4 + 1, or 2 + 3, or 1 + 4, or 3 + 2.

4 may be renamed:

3 + 1, or 1 + 3, or 2 + 2.

3 may be renamed:

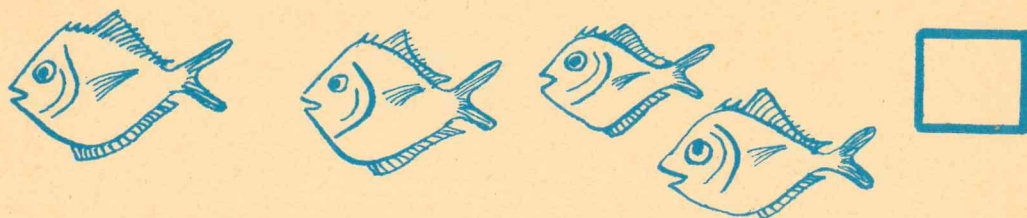
2 + 1, or 1 + 2.

2 may be renamed:

1 + 1.

EQUALS SIGN EQUATIONS

Write the cardinal number of the set of fish in the number frame.



Separate the set of fish into two subsets, and rename the number of the set.

$$\boxed{3} + \bigcirc$$

$3 + 1$ stands for the same number as 4.

We say that $3 + 1$ *equals* 4.

The equals sign is written $=$

We write the number sentence like this: $3+1=4$.

We read it,

“Three plus one equals four.”

A number sentence using an equals sign is called an *equation*.

In an equation, the numerals on both sides of the equals sign name the same number.

Write the numerals in the number frames so each equation is true.

$3+1=\square$

$2+1=\square$

$1+1=\square$

$1+3=\square$

$1+2=\square$

$2+2=\square$

NOTE: The set of 4 fish above can be separated into subsets of $3 + 1$. The concept of addition facts as equations paves the way for algebra, and makes addition more meaningful.

RENAMING NUMBERS TO MAKE EQUATIONS

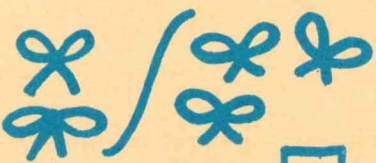
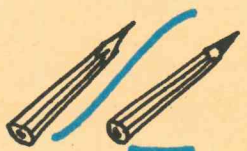
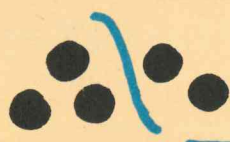
An equation is one kind of number sentence.

It is a sentence written with numerals and signs.

The number frame in an equation is called a *placeholder* because it holds the place for a numeral which will make the equation true.

The sets below are separated into subsets.

Write the numeral in the placeholder in the equation, to make each number sentence true.

 $3 + \square = 4$	 $2 + 3 = \square$
 $\square + 1 = 5$	 $1 + \square = 2$
 $3 + \square = 5$	 $2 + \square = 4$



NOTE: The child should realize that the numeral or numerals on one side of the equals sign stands for the same number of objects as the numeral or numerals on the other side. They both name the same set.

Addition facts should be learned so that any missing numeral can be filled in to make a true equation.

JOINING SETS TO MAKE EQUATIONS

Sets can be separated into subsets.

Fill in the placeholders to make true equations.



Sets can be joined together, too.

In the number frame below, write the cardinal number of the set of lizards.



In the number frame below, write the cardinal number of the set of turtles.



When we join the set of lizards to the set of turtles, we get a new set.



Write the numeral for the new set.

We can write an equation about joining the two sets.

$$3+2=\square$$

We can write an equation about separating a set of 5 things into 2 subsets, or about joining two sets to make a new set with five elements.

NOTE: It should be noted that an equation can be made either by renaming a number, or about joining sets.

IS GREATER THAN IS LESS THAN

Draw a line around the set in each block which has the greater number of members.

•• ▲▲	✓ ★★	○○○ □□
AAA BBBB		☺☺☺ ◎◎

We say that the larger set is *greater than* the smaller set.

This is the sign for *is greater than*: $>$

Finish these number sentences so they are true:

$$\begin{array}{ccc} 5 > \square & 3 > \square & 5 > \square \\ 4 > \square & 2 > \square & 3 > \square \end{array}$$

Draw a line around the set in each block which has fewer members.

↑↑↑ 888	AAA BB	1
⊙⊙ ✕	○○○ ♀♀	☙☙☙ ☺☺☺

We say that the smaller set is *less than* the larger set.

This is the sign for *is less than*: $<$

Finish these number sentences so they are true:

$$\begin{array}{ccc} 1 < \square & 3 < \square & 4 < \square \\ 2 < \square & 4 < \square & 3 < \square \end{array}$$

NOTE: Number sentences may be made by using the sign $>$ (is greater than) or the sign $<$ (is less than). Sentences using these signs show the relationship between numbers. More than one number can make these sentences true.

INEQUALITIES

A number sentence which uses the sign
is greater than $>$
or *is less than* $<$
is called an *inequality*.

The numerals on each side of the inequality sign do not rename the same number. They do not stand for the same number of objects.

Write the signs of inequality to make each number sentence true.

5	$>$	4	:	1	2	:	5	3
4		3	:	2	3	:	4	2
3		2	:	3	4	:	1	5
2		1	:	4	5	:	2	3

REVIEW OF EQUATIONS

Inequalities name numbers which are not equal.

Equations name numbers which are equal.

Make each equation true.

$3 + \square = 5$

$\square + 3 = 4$

$2 + 3 = \square$

$2 + 2 = \square$

$2 + \square = 3$

$4 + \square = 5$

$1 + \square = 5$

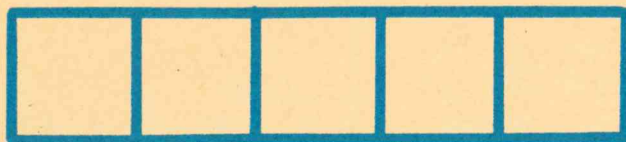
$\square + 1 = 2$

$1 + \square = 3$

NOTE: The child should understand that the numerals on each side of the sign of equality or inequality stand for sets of that many objects.

SIX SEVEN

How many squares are in the strip below?

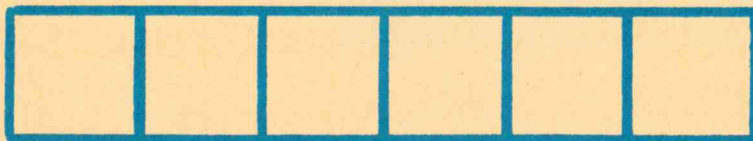


Starting at the left, write the numerals from 1 to 5.
Draw another strip with 5 squares in the space below, and
write the numerals.

Next to the square with the numeral 5, draw one more square.
Now there are *six* squares.
The numeral for six is written 6.
Write 6 in the new square above.

6

How many squares are in the strip below?



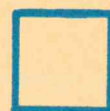
Starting at the left, write the numerals from 1 to 6.
Next to the square with the numeral 6, draw one more square.
Now there are *seven* squares.
The numeral for seven is written 7.
Write 7 in the new square.

7

NOTE: This page introduces the numerals 6 and 7. Each strip is a set with that many squares. The child should realize that 6 is 1 more than 5, and 7 is 1 more than 6. If further practice is needed, have the child collect, or draw, other sets with 6 elements and with 7 elements.

ADDITION FACTS FOR SIX AND SEVEN

Write the cardinal number of the set of bugs in the number frame.



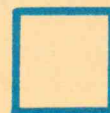
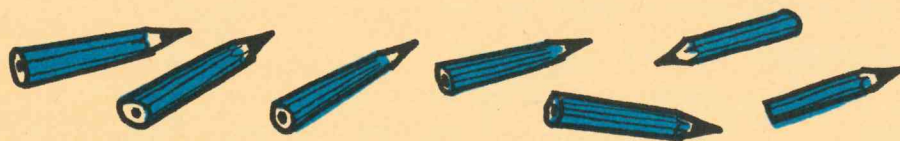
Separate the set into two subsets.

How many equations can you write about separating the set of 6 bugs into different subsets?

Each time you make one of the equations true, you are re-naming 6.

$$\begin{array}{lll} 1 + \square = 6 & 3 + \square = 6 & 5 + \square = 6 \\ 2 + \square = 6 & 4 + \square = 6 & \end{array}$$

Write the cardinal number of the set of pencils in the number frame.



Separate the set into two subsets.

How many equations can you make about separating the set of 7 pencils into different subsets?

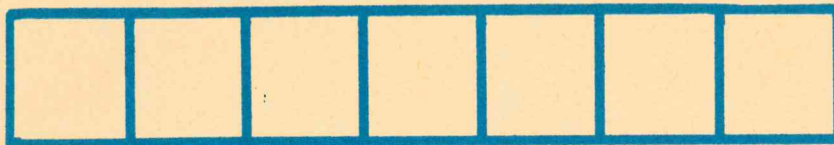
Each time you make one of the equations true, you are re-naming 7.

$$\begin{array}{lll} 1 + \square = 7 & 4 + \square = 7 & 5 + \square = 7 \\ 6 + \square = 7 & 3 + \square = 7 & 2 + \square = 7 \end{array}$$

NOTE: Here are all the addition facts for 6 and 7. The child should understand that he is forming the equations by renaming the number.

EIGHT NINE

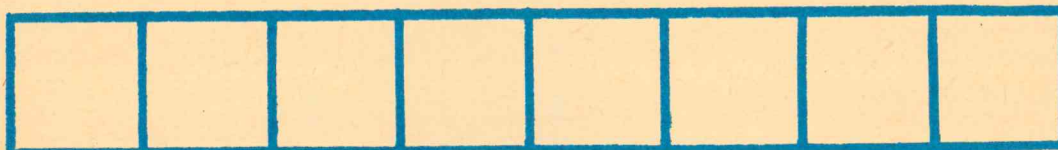
How many squares are in the strip below?



Starting at the left, write the numerals from 1 to 7.
Draw another strip with 7 squares in the space below and
write the numerals.

Next to the square with the numeral 7, draw one more square.
Now there are *eight* squares.
The numeral eight is written 8.
Write 8 in the new square.
How many squares are in the strip below?

8



Next to the square with the numeral 8, draw one more square.
Now there are *nine* squares.
The numeral for nine is written 9.
Write 9 in the new square.

9

NOTE: This page introduces the numerals 8 and 9.

The 8-strip is a set of 8 squares.

The 9-strip is a set of 9 squares.

If further practice is needed, have the child collect, or draw, others sets with 8 and 9 members.

ADDITION FACTS FOR EIGHT AND NINE

Write the cardinal number of the set of umbrellas in the number frame.



Separate the set of umbrellas into any two subsets.

How many equations can you write about separating the set of 8 umbrellas into different subsets?

Each time you make one of the equations true, you are re-naming 8.

$$1 + \square = 8$$

$$4 + \square = 8$$

$$7 + \square = 8$$

$$2 + \square = 8$$

$$5 + \square = 8$$

$$3 + \square = 8$$

$$6 + \square = 8$$

Write the cardinal number of the set of flags in the number frame.



Separate the set of flags into any two subsets.

How many equations can you write about separating the set of 9 flags into different subsets?

Each time you make one of the equations true, you are re-naming 9.

$$1 + \square = 9$$

$$4 + \square = 9$$

$$1 + \square = 9$$

$$2 + \square = 9$$

$$5 + \square = 9$$

$$8 + \square = 9$$

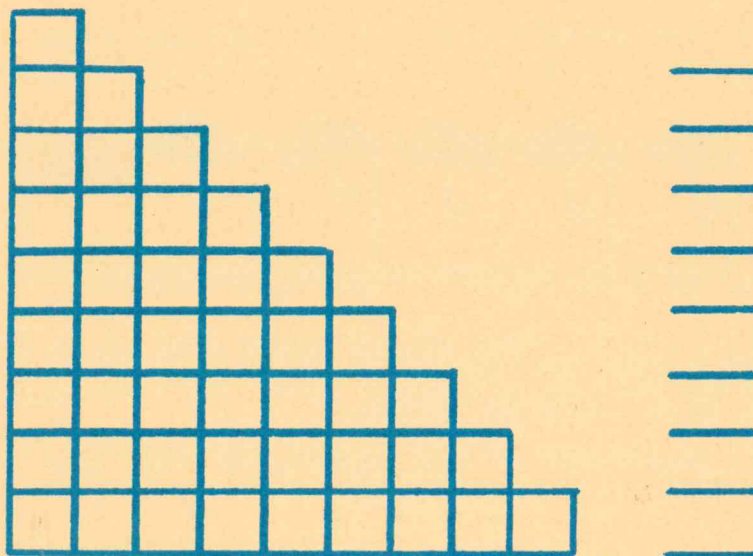
$$3 + \square = 9$$

$$6 + \square = 9$$

NOTE: Here are all the addition facts for 8 and 9. The child should realize that he is renaming a number of objects in the set when he makes each equation true.

COUNTING NUMBERS TO NINE

Starting with 1, write the numerals to show counting.



Next to each row, write the cardinal number of the set of squares in the row.

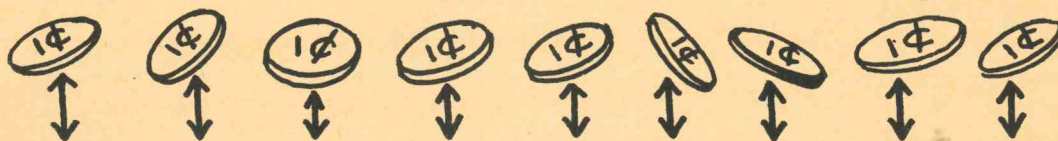
The last numeral in the row tells the cardinal number of that set of squares.

Each row has one more square than the one above it.

Each row has one more numeral than the one above it.

Now you know the first nine numbers in the set of counting numbers.

Match the set of counting numbers with the set of pennies.



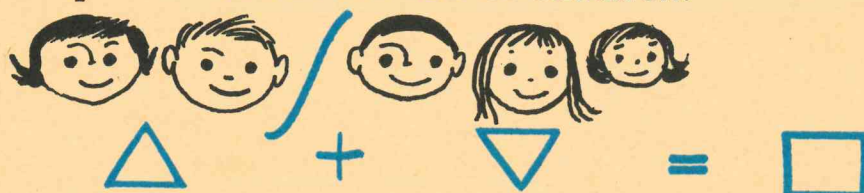
Write the numerals on the lines.

Write the first nine numerals in the set of counting numbers.

NOTE: The child has now learned to read and write numerals to nine and to count sets with up to nine elements. He has also learned the set of counting numbers to nine, and all the addition facts to nine.

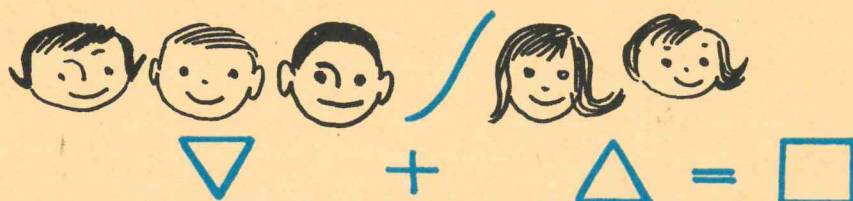
CHANGING THE ORDER IN ADDITION

Write the equation about the sets of children.



The numbers which are added together are called *addends*.
The result is called the *sum*.

Now change the position of the sets and write another equation.



No matter which addend is first, the sum of the addends is the same.

Make each equation true.

$2+1=\square$	$4+1=\square$	$3+1=\square$
$1+2=\square$	$1+4=\square$	$1+3=\square$

In each pair of equations, only the order of the addends has been changed. But the sum in each pair is the same.

This is true of all addition.

The order in which sets are joined does not change the number of elements in the new set.

The order in which addends are written does not change the sum.

NOTE: This page introduces one of the eleven properties of arithmetic which can be used in any branch of mathematics. Knowledge of these properties gives the child a much clearer understanding of what the process of addition is.

COMMUTATIVE PROPERTY OF ADDITION

Read each of the following sentences.

If it is true, mark it T.

If it is false, mark it F.

$2+1$ names the same number as $1+2$.
 $3+2$ names the same number as $2+3$.
 $4+1$ names the same number as $1+4$.

The sentences above are all true.

We have said that $=$ is the sign for "equals."

Make the equations true.

$$2+1 = 1 + \square$$
$$3+1 = 1 + \square$$

$$3+2 = 2 + \square$$
$$4+1 = 1 + \square$$

The order in which the addends are written does not make any difference.

This is called the *commutative property of addition*.

Commutative is pronounced *come-you-ta-tiv*.

When you commute on a train, you go from one place to another; then you come back.

Finish the following equations to show the commutative property of addition.

$$5+1 = \square + \square$$
$$4+2 = \square + \square$$
$$3+4 = \square + \square$$

$$5+2 = \square + \square$$
$$6+1 = \square + \square$$
$$6+2 = \square + \square$$

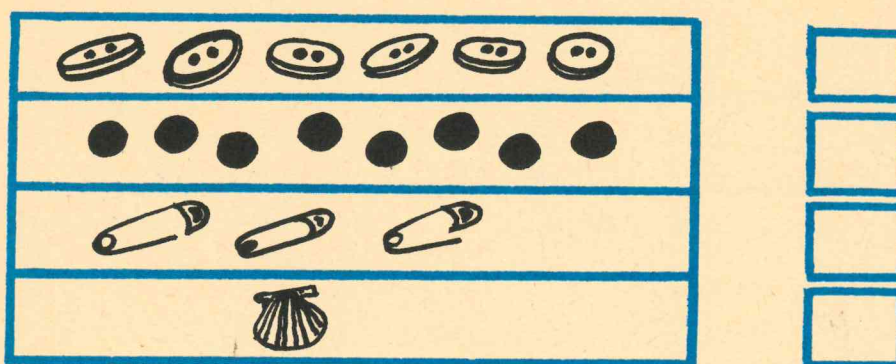
NOTE: The commutative property of addition is usually written $a + b = b + a$.
The child should understand this concept, although he does not have to know the formula for it.

EMPTY SET

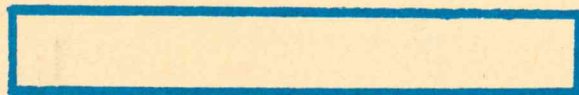
ZERO

How many elements are in each set below?

Write the cardinal number of the set in the number frame next to the set.



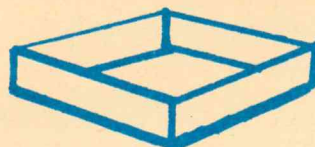
How many elements are in this set below?



There are no elements in this set.

It is called an *empty set*.

When there is nothing in a box, we say it is an empty box.



When there is no food on a dish, we say it is an empty dish.



When there are no objects in a set, we say it is an *empty set*.

The number name for the empty set is *zero*.

The numeral for zero is written 0.

Practice writing the numeral 0 below.

0

NOTE: In reading 0, always call it *zero*, never *oh* or *nothing*.

Zero is not nothing; it names the empty set.

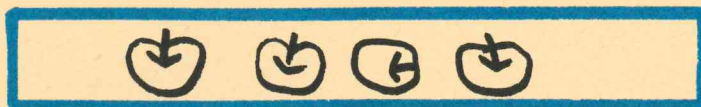
Have the child give you other examples of empty sets; for instance:

the number of purple cows

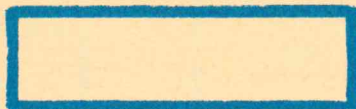
the number of people who have flown to Mars

ZERO AS AN ADDEND

Write the numeral for the set.



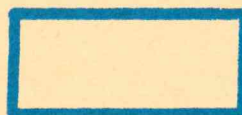
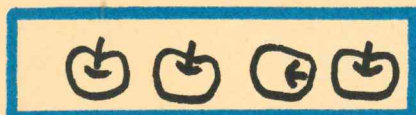
Write the numeral for the set.



The second set is an empty set.

Join the first set and the empty set.

Write the equation about joining the sets.



+



=



When zero is added to another number, the sum is always the other number.

Make these equations true.

$$5 + 0 = \square$$

$$8 + 0 = \square$$

$$3 + 0 = \square$$

$$7 + 0 = \square$$

$$9 + 0 = \square$$

$$2 + 0 = \square$$

Make these equations true.

$$3 + 2 = \square$$

$$4 + 1 = \square$$

$$\square + 2 = 3$$

$$2 + \square = 4$$

$$1 + 4 = \square$$

$$3 + 0 = \square$$

$$2 + \square = 2$$

$$1 + \square = 1$$

$$4 + \square = 4$$

NOTE: The child should realize that a set with no objects in it is called an empty set. He should associate the term *empty set* with zero.

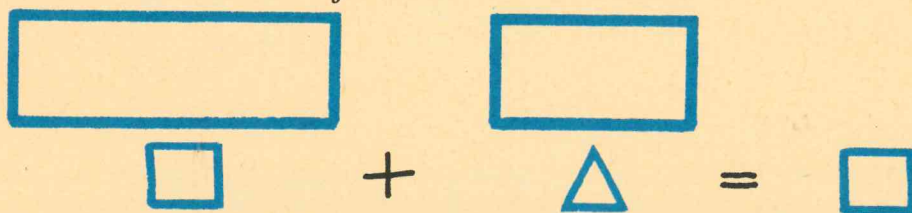
THE IDENTITY ELEMENT OF ZERO IN ADDITION

When a set has an empty set joined to it, the number of members in the new set is the same as the number of members in the first set.

Draw a set of 3 things in one block.

Draw an empty set in the other block.

Then write the equation to show the number of the new set when the sets are joined.



Zero is called the *identity element* of addition.

When zero is an addend, the sum is the same as it was before adding the zero.

Make these equations true.

$$\begin{array}{ll} 3+2=2+\square & 1+3=\square \\ 4+0=\square & 0+4=\square \\ 3+\square=5 & 4+1=1+\square \end{array}$$

If the equation shows the identity property of zero in addition, mark it with an I.

If the equation shows the commutative property of addition, mark it with a C.

NOTE: The child has now been introduced to two of the *properties* of arithmetic:

1. the commutative property.
2. the identity property of zero.

Have the child give other examples of both properties.

The placeholders above should be filled in like this:

$$\begin{array}{ll} 3+2=2+3 & \text{C} & 1+3=4 \\ 4+0=4 & \text{I} & 0+4=4 & \text{I} \\ 3+2=5 & & 4+1=1+4 & \text{C} \end{array}$$

OPEN SENTENCES

An equation is a number sentence which names the same number on both sides of the equals sign.

An inequality is a number sentence which names numbers that are not equal.

Make each equation true.

$$\begin{array}{l} 4 + 3 = \square \\ 3 + \square = 8 \end{array}$$

$$\begin{array}{l} 6 + \square = 7 \\ \square + 4 = 5 \end{array}$$

$$\begin{array}{l} 8 + 1 = \square \\ \square + 5 = 7 \end{array}$$

Make each inequality true.

$$\begin{array}{l} 5 > \square \\ 8 > \square \end{array}$$

$$\begin{array}{l} 4 < \square \\ 6 < \square \end{array}$$

$$\begin{array}{l} 7 < \square \\ 9 > \square \end{array}$$

An equation or inequality that does not have numerals in all the placeholders is called an *open sentence*.

We cannot tell whether the sentence is true or false until numerals have been written in the placeholders.

Look at each number sentence below.

If the numeral in the placeholder makes it true, mark it with this sign. ✓

If the numeral in the placeholder does not make it true, mark it with this sign. ✗

$$3 + \boxed{4} = 7$$

$$2 < \boxed{5}$$

$$1 + 4 = 4 + \boxed{2}$$

$$5 + \boxed{3} = 2 + 5$$

$$4 + 0 = \boxed{5}$$

$$7 > \boxed{5}$$

$$\boxed{3} + 6 = 8$$

$$8 > \boxed{7}$$

$$\boxed{0} + 5 = 5$$

NOTE: A number sentence is not necessarily true. If any part of it is false, the whole sentence is false.

The concept of an *open sentence* is preparing the way for the study of higher mathematics, and keeps arithmetic from being a subject unrelated to other forms of mathematics.

VERTICAL NOTATION

Until now, we have been writing all our addition facts in *rows*, like this:

$$1+3=4 \quad 3+1=\square \quad 2+2=\square$$

There is another way addition facts can be written.

We can write the same addition facts this way:

$$\begin{array}{r} 1 \\ +3 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 3 \\ +1 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 2 \\ +2 \\ \hline 4 \end{array}$$

We write the addends one under the other, in a *column*.

This is called *vertical notation*.

The line under the second addend can be read "is equal to" or "equals," just like the equals sign =.

Make these equations true.

$$3+2=\square \quad 4+1=\square \quad 1+4=\square$$

Now the same equations are written the new way. Write each *sum* under the *addends*.

$$\begin{array}{r} 3 \\ +2 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ +1 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ +4 \\ \hline \end{array}$$

Notice that in the new way placeholders are not used. To finish the number fact, we always write the sum under the line which says "equals."

NOTE: The child should realize that the addition facts in vertical notation are no different from the addition facts written as equations.

He should understand that the numerals stand for a number of objects, just as they do in equations.

VERTICAL NOTATION TO NINE

Make these addition exercises true.

$$\begin{array}{r} 1 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +2 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +5 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +6 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ +2 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +5 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +7 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ +2 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +6 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +5 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +8 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +7 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +6 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ +5 \\ \hline \end{array}$$

NOTE: Here are the addition facts up to nine written in vertical notation. Addition facts using zero as an addend have not been included here.

VERTICAL NOTATION ZERO AS AN ADDEND

Make these addition exercises true.

$$\begin{array}{r} 1 \\ + 0 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ + 0 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ + 0 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ + 0 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ + 0 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ + 0 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ + 0 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 0 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ + 0 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ + 1 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ + 2 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ + 3 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ + 4 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ + 5 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ + 6 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ + 9 \\ \hline \end{array}$$

$$5 + \square = 5$$

$$\square + 6 = 6$$

$$4 + 0 = \square$$

$$0 + 9 = \square$$

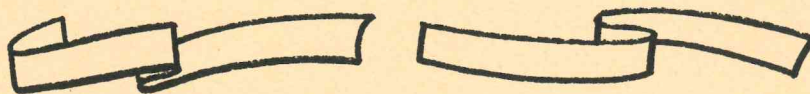
In each of these addition facts, one of the addends is zero. Does it make any difference whether the zero is the first or the second addend? Yes ☐ No ☐

What is the name of the arithmetic property which tells about using zero as an addend?

NOTE: The identity property of zero for addition is just as true in vertical notation as in writing equations. Stress the fact that whether zero is the first or the second addend, the sum is the same numeral as the other addend.

UNDO

Suppose you have two ribbons.



Color the first ribbon blue.

Color the second ribbon red.

Write the numeral for the set of blue ribbons.



Write the numeral for the set of red ribbons.



Join the ribbons.



Finish the equation.

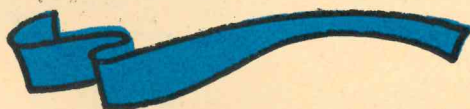
$$1 + \square = \square$$

You now have 2 ribbons, joined together.

How can you separate them?

You can *undo* the knot.

If you take away the one red ribbon, how many ribbons are left?



Only the one blue ribbon is left.

When you *undo* the knot, you are separating the set into subsets.

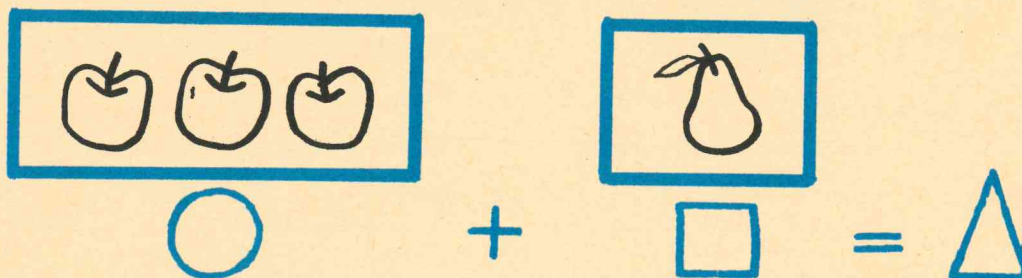
NOTE: Be sure the child understands clearly the operation of joining things, and then “undoing” the operation of joining. This will lead to the concept of subtraction.

UNION OF SETS

TAKE AWAY

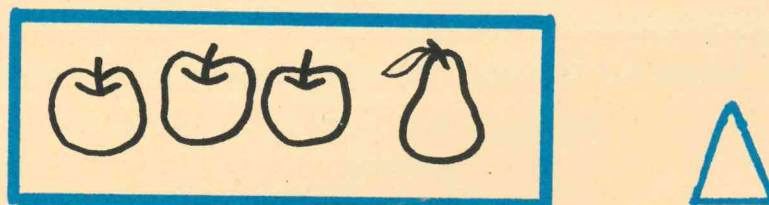
There are two sets below.

Write the equation which shows the numbers of the two sets, and the number of the new set.



When sets are joined, the operation is called the *union* of sets. When numbers are added, the operation is called *addition*.

Here is a set of fruit.



Write the cardinal number of the set in the number frame. Now pretend that you want to eat the pear.

Draw a line around it, to show that you want to *take away* the pear. You are separating the sets into subsets. You are doing the same thing you did when you untied the ribbon. Write the numeral that shows how many things you did not take away.



NOTE: The term *take away* is used only in discussing sets, for the physical operation of taking away. The operation of taking away is the same as separating the set into subsets. Sometimes "*remove*" may be used in referring to set separation.

MINUS

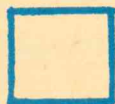
Write the numeral for the set of hats in the first number frame.



$$\square + \triangle = \bigcirc$$

Draw a clown hat next to the two hats above.
Finish the equation to show the number of hats you drew,
and the number in the new set.

Now *undo* the operation of joining the two sets.
Take away the clown hat by drawing a line around it.
You are separating the set of hats into subsets.
Write the numeral for the set of hats you took away.



How many hats are left?

We can write an equation for this operation too. It tells that
you had a set of three hats, then took away a subset of one hat;
and a subset of two hats was left. In numerals, the equation
looks like this.

$$3 - 1 = 2$$

We read this equation: three *minus* one equals two.
The little line — between the 3 and the 1 is always read
minus.

NOTE: The equation for the set of hats should be $2 + 1 = 3$.
The terms *take away* or *remove* are used in discussing sets.
The term *minus* is used only in discussing numerals.

SUBTRACTION



Write the numeral for the set of stars. 

Next to the set, draw a set of two more stars.

Write the numeral for the set of stars you drew. 

Write the equation to show the number of stars in all.

$$\triangle + \square = \bigcirc$$

Draw another set of three stars.

Write the numeral for the set of stars. 

Now take away a subset of two stars.

Write the numeral for the subset you took away. 

How many stars are left? 

Write the equation to show what you did.

$$3 - \square = \triangle$$

When you take away part of a set, you are separating it into subsets.

The minus sign tells that a set is taken away.

It tells that a numeral is *subtracted*.

The name of this operation is *subtraction*.

Make these subtraction equations true.

$$3 - 1 = \square$$

$$3 - 2 = \triangle$$

NOTE: Have the child work with sets of 2 or 3 objects.

Have him take away a subset, and then write the subtraction equation to show what he has done. He may write any of these equations:

$$3 - 2 = 1$$

$$3 - 1 = 2$$

$$2 - 1 = 1$$

Be sure the sign for subtraction is read *minus*.

DIFFERENCE

When one number is added to another, the result is called the *sum*.

Write the sum for each equation.

$$3 + 3 = \square$$

$$4 + 4 = \square$$

$$4 + 5 = \square$$

$$6 + 3 = \square$$

$$7 + 1 = \square$$

$$3 + 4 = \square$$

When one number is subtracted from another, the result is called the *difference*.

Write the differences in the following subtraction equations:

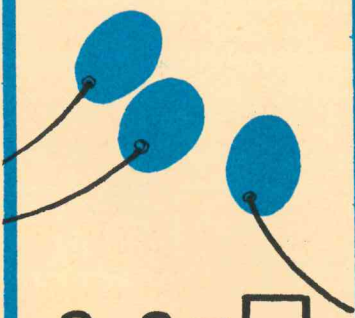
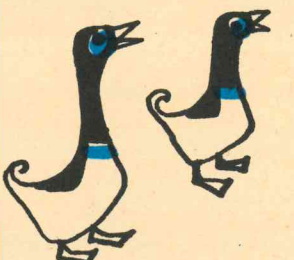
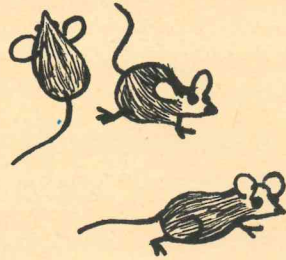
$$3 - 2 = \square$$

$$2 - 1 = \square$$

$$3 - 1 = \square$$

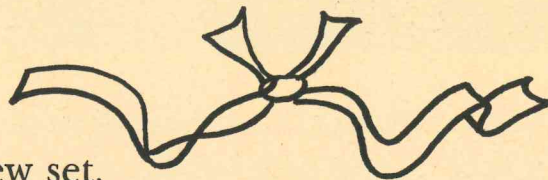
Draw a line to show two subsets within each set.

Write the difference for each equation.

 $3 - 2 = \square$ $3 - 1 = \square$	 $2 - 1 = \square$	 $3 - 1 = \square$ $3 - 2 = \square$
--	--	--

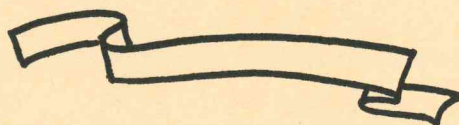
NOTE: This page strengthens the concept that subtraction is the operation which shows set separation.

INVERSE



You can join sets to make a new set.

You can separate a set into subsets and take away one of the subsets.



First you put sets together.

Then you take them apart.

When you take sets apart, you are *undoing* what you have put together.

An addition equation uses numerals to show what is added.

$$2+1=3$$

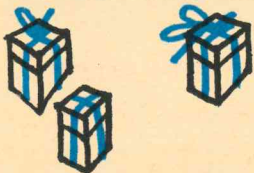
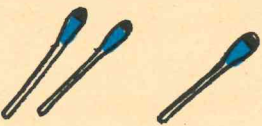
A subtraction equation uses numerals to show what is subtracted.

$$3-2=1$$

We can say that in a subtraction equation, we *undo* an addition equation.

Subtraction is the *inverse* of addition.

Make these equations true.

		
$2+1=\square$ $3-1=\square$	$2+\square=3$ $3-\square=2$	$\square+2=3$ $3-\square=1$

NOTE: The concept of *inverse operation* is very important in modern mathematics. Subtraction facts should be learned as the inverse, or opposite, of addition facts.

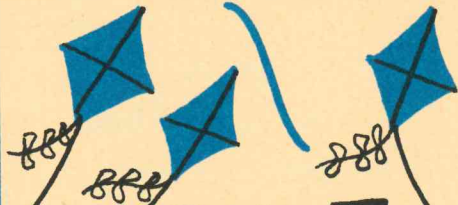
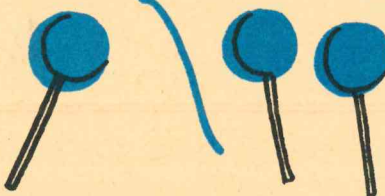
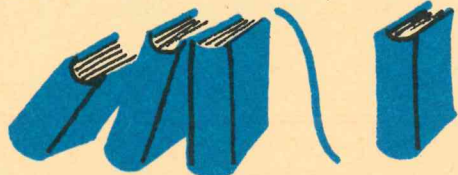
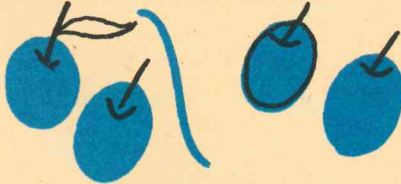
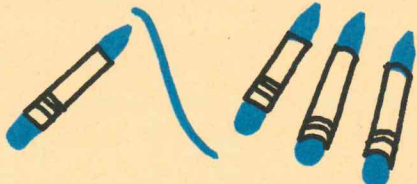
SUBTRACTION FACTS TO FOUR

Subtraction is the inverse or undoing of addition.

Look at the sets below.

Write the sum for each addition equation.

Write the difference for each subtraction equation.

 $1 + 1 = \square$ $2 - 1 = \triangle$	 $2 + 1 = \square$ $3 - 1 = \triangle$
 $1 + 2 = \square$ $3 - 2 = \triangle$	 $3 + 1 = \square$ $4 - 1 = \triangle$
 $2 + 2 = \square$ $4 - 2 = \triangle$	 $1 + 3 = \square$ $4 - 3 = \triangle$

NOTE: The child should understand that the addition equation for each set shows two sets that have been joined.

The subtraction equation for each set separates the set into subsets again. It undoes the addition equation. Lead the child to see that the number of the set with which the addition equation began is the number of the set with which the subtraction equation ends.

SUBTRACTION FACTS FOR FIVE

Write the sum for each addition equation.

Write the difference for each subtraction equation.

 $4 + 1 = \square$ $5 - 1 = \triangle$	 $3 + 2 = \square$ $5 - 2 = \triangle$
 $2 + 3 = \square$ $5 - 3 = \triangle$	 $1 + 4 = \square$ $5 - 4 = \triangle$

If you think of the sets above, and the number sentences that are with them, you will find it easy to fill in the placeholders below.

$$4 + \square = 5$$

$$\square + 3 = 5$$

$$2 + \square = 5$$

$$1 + 4 = \square$$

$$5 - 1 = \square$$

$$5 - \square = 4$$

$$\square - 3 = 2$$

$$5 - \square = 1$$

NOTE: The child by now should understand the connection between the related addition and subtraction facts.

In the equations at the bottom of the page, the number frame can be read either "placeholder" or "some number." For example:

$4 + \square = 5$ would be read:

four plus placeholder equals five or

four plus some number equals five.

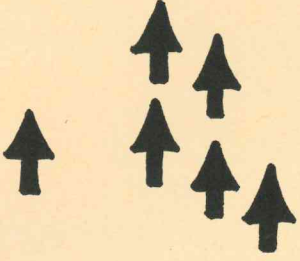
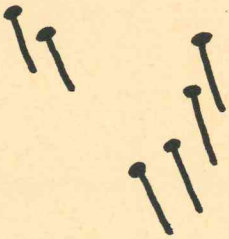
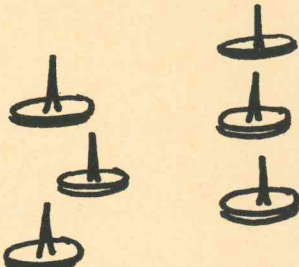
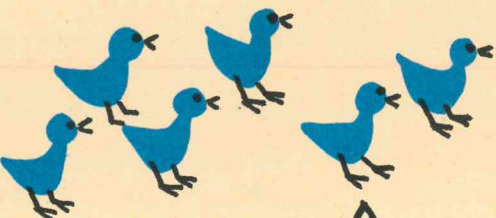
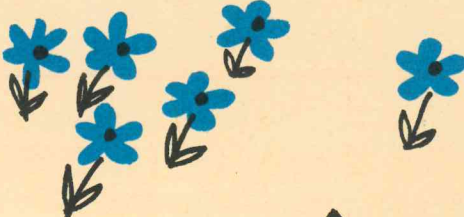
In a subtraction equation, such as $5 - \square = 4$, the child is actually being asked to find the number which when added to 4, gives 5.

In other words, the placeholder is a missing addend, because when 1 is added to 4, the sum is 5.

SUBTRACTION FACTS FOR SIX

Write the sum for each addition equation.

Write the difference for each subtraction equation.

 $1 + 5 = \triangle$ $6 - 5 = \square$	 $2 + 4 = \triangle$ $6 - 4 = \square$	 $3 + 3 = \triangle$ $6 - 3 = \bigcirc$
 $4 + 2 = \triangle$ $6 - 2 = \square$	 $5 + 1 = \triangle$ $6 - 1 = \nabla$	

Write the numerals in the open sentences to make them true.

$$6 - \square = 1$$

$$3 + \square = 6$$

$$6 - \square = 5$$

$$6 - 4 = \square$$

$$5 + \square = 6$$

$$6 - 2 = \square$$

$$6 - \square = 3$$

$$\square + 2 = 6$$

$$6 - 3 = \square$$

NOTE: If necessary, review the work on open sentences on page 28.

VERTICAL NOTATION IN SUBTRACTION

We learned that addition facts do not always have to be written in a row. They can be written in a column. Find the sums.

$$\begin{array}{r} 4 \\ + 2 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ + 1 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ + 3 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ + 1 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ + 4 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ + 2 \\ \hline \end{array}$$

Subtraction facts can be written in the same form. The minus sign goes in front of the second numeral. The line below the numerals is read "equals."

$$\begin{array}{r} 6 \\ - 2 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ - 3 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ - 2 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ - 4 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ - 3 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ - 2 \\ \hline \end{array}$$

When subtraction facts are written in this form, a placeholder is not needed. Remember, however, that the numerals on one side of the line must have the same value as the numeral on the other side of the line, just as they do in an equation.

$$\begin{array}{r} 6 \\ - 2 \\ \hline 4 \end{array}$$

$$6 - 2 = \square$$

NOTE: The vertical form of notation becomes necessary when numerals are larger than 10. This page gives practice in preparation for the use of 2-digit numerals.

ZERO IN SUBTRACTION

Do you remember what happened when the empty set was joined to a set?

Join the sets below.



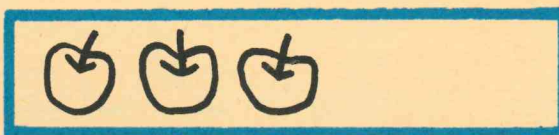
Finish the equation.

$$3 + 0 = \square$$

We said that zero is the *identity element* for addition, because it does not change the number to which it is added.

What happens to a set when the empty set is taken away, or separated, from a set?

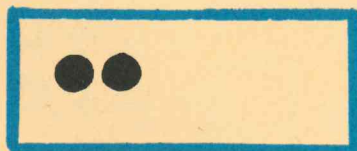
Separate the set below into a subset of three and the empty set.



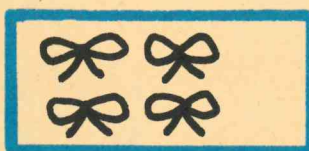
Finish the equation.

$$3 - 0 = \square$$

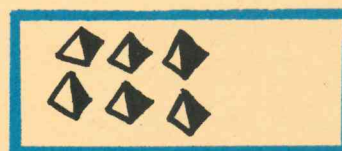
When zero is subtracted from another number, the other number does not change.



$$2 - 0 = \square$$



$$4 - 0 = \square$$



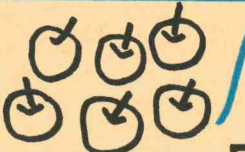
$$6 - 0 = \square$$

NOTE: Have the child practice taking away an empty set from a given set and then writing the subtraction fact for what he did, until it is quite clear to him that subtraction with zero does not change the given number.

ADDITION AND SUBTRACTION WITH ZERO

Make each equation true.

In each box, draw the number of elements needed to show the missing subset.

 $2 + 0 = \square$ $2 - 0 = \square$	 $4 + 0 = \square$ $4 - 0 = \square$	 $6 + 0 = \square$ $6 - 0 = \square$
 $1 + 0 = \square$ $1 - 0 = \square$	 $3 + 0 = \square$ $3 - 0 = \square$	 $5 + 0 = \square$ $5 - 0 = \square$

The sum of any number plus zero equals that number.

The difference when any number has zero subtracted from it is that number.

REVIEW OF SUBTRACTION FACTS TO SIX

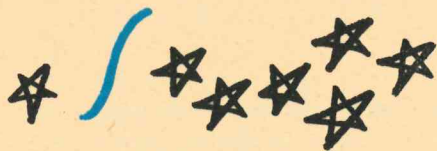
Find the differences for the following subtraction exercises.

$\begin{array}{r} 6 \\ -3 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ -1 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ -5 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ -2 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ -1 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ -3 \\ \hline \end{array}$
$\begin{array}{r} 3 \\ -2 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ -2 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ -3 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ -0 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ -1 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ -4 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ -1 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ -4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ -0 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ -2 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ -1 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ -6 \\ \hline \end{array}$

SUBTRACTION FACTS FOR SEVEN

Write the sum for each addition equation.

Write the difference for each subtraction equation.



$$1 + 6 = \square$$

$$7 - 6 = \square$$



$$2 + 5 = \square$$

$$7 - 5 = \square$$



$$3 + 4 = \square$$

$$7 - 4 = \square$$



$$4 + 3 = \square$$

$$7 - 3 = \square$$



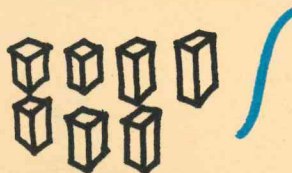
$$5 + 2 = \square$$

$$7 - 2 = \square$$



$$6 + 1 = \square$$

$$7 - 1 = \square$$



$$7 + 0 = \square$$

$$7 - 0 = \square$$

Make each equation true.

$$7 - \square = 2 \quad 7 - \square = 6$$

$$7 - 6 = \square \quad 7 - \square = 7$$

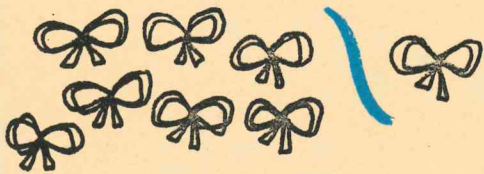
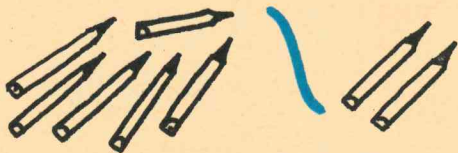
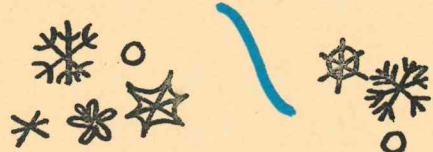
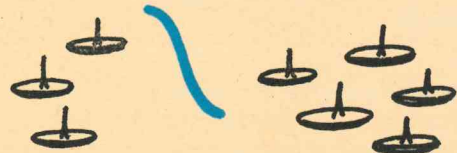
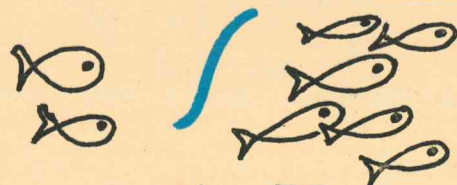
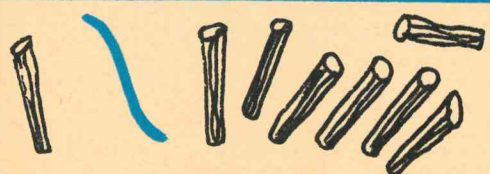
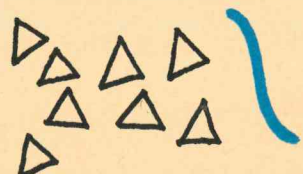
$$7 - 4 = \square \quad 7 - 3 = \square$$

NOTE: This page introduces the subtraction facts for seven. The child should understand the relationship between joining two sets to make seven members, and separating the set into subsets and taking away one subset.

SUBTRACTION FACTS FOR EIGHT

Write the sum for each addition equation.

Write the difference for each subtraction equation.

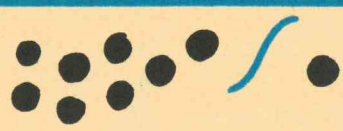
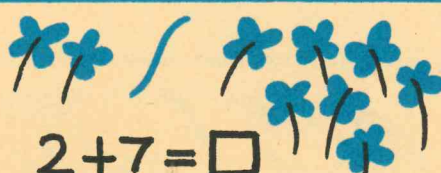
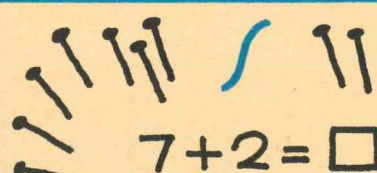
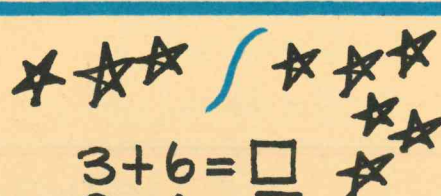
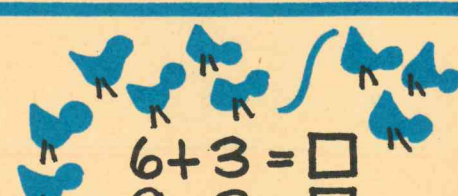
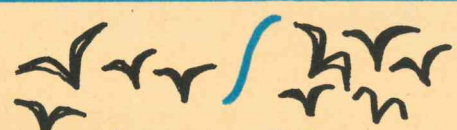
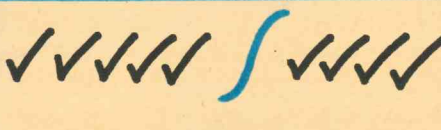
 $7 + 1 = \square$ $8 - 1 = \square$	 $6 + 2 = \square$ $8 - 2 = \square$
 $5 + 3 = \square$ $8 - 3 = \square$	 $4 + 4 = \square$ $8 - 4 = \square$
 $3 + 5 = \square$ $8 - 5 = \square$	 $2 + 6 = \square$ $8 - 6 = \square$
 $1 + 7 = \square$ $8 - 7 = \square$	 $8 + 0 = \square$ $8 - 0 = \square$

NOTE: This page introduces the subtraction facts for 8. The child should understand that the subtraction operation is the inverse of the addition operation.

SUBTRACTION FACTS FOR NINE

Write the sum for each addition equation.

Write the difference for each subtraction equation.

 $1 + 8 = \square$ $9 - 8 = \square$	 $8 + 1 = \square$ $9 - 1 = \square$
 $2 + 7 = \square$ $9 - 7 = \square$	 $2 + 7 = \square$ $9 - 2 = \square$
 $3 + 6 = \square$ $9 - 6 = \square$	 $3 + 6 = \square$ $9 - 3 = \square$
 $4 + 5 = \square$ $9 - 5 = \square$	 $5 + 4 = \square$ $9 - 4 = \square$

Look at the two addition equations on each row above.

What property of addition do they show?

There are two more subtraction facts for nine.

$$9 + 0 = \square \quad 9 - 0 = \square$$

Make the equations true.

NOTE: The addition property shown is the commutative property

$8 + 1 = 9$; $1 + 8 = 9$, etc.

If the child does not see this immediately, point out to him that in the sets shown in each row, only the order in which the sets have been joined is changed.

SUBTRACTING A NUMBER FROM ITSELF

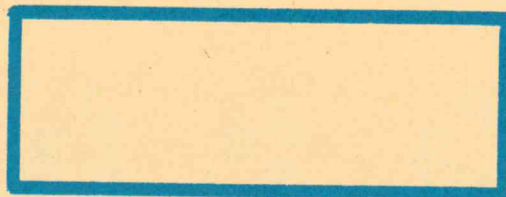
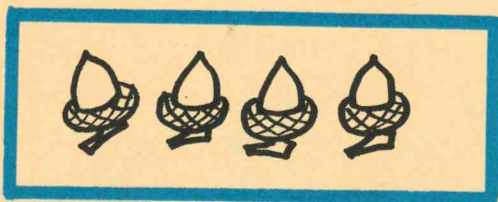
Make each equation true.

$$\begin{array}{l} 7 - 0 = \square \\ 6 - 0 = \square \end{array}$$

$$\begin{array}{l} 5 - \square = 5 \\ 4 - \square = 4 \end{array}$$

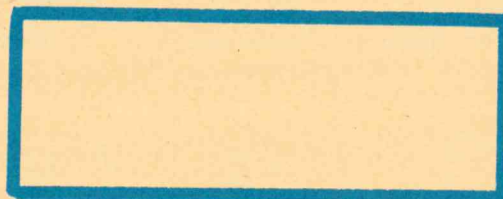
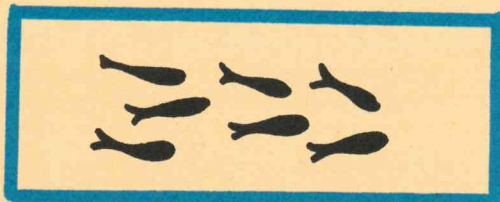
We have said that when zero is subtracted from a number, the given number is the difference.

Suppose we want to take a whole set away from itself. That means, if we have a set of 4 members, we want to take 4 members away.



Finish the equation: $4 - 4 = \square$ Show how many are left.

Suppose we have a set of 7 members, and we want to take 7 members away.



Show how many are left.

Finish the equation: $7 - 7 = \square$

Make each equation true.

$$9 - 9 = \square$$

$$8 - 8 = \square$$

$$3 - 3 = \square$$

$$5 - \square = 0$$

$$6 - \square = 6$$

$$2 - \square = 0$$

NOTE: The child should be led to realize that any number is a subset of itself; that is, it can be separated into a set and an empty set.

REVIEW OF ADDITION FACTS TO NINE

Finish the addition facts.

$$\begin{array}{r} 1 \\ +6 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ +5 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +8 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +7 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +5 \\ \hline \end{array}$$

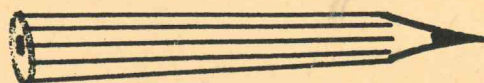
$$\begin{array}{r} 1 \\ +0 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +5 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ +0 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +3 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +6 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ +0 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +6 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ +1 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ +3 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ +0 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ +0 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +0 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +5 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +0 \\ \hline \end{array}$$



REVIEW OF SUBTRACTION FACTS TO NINE

Finish the subtraction facts.

$$\begin{array}{r} 9 \\ -5 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ -7 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ -0 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ -0 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ -6 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ -6 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -3 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ -6 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ -8 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ -5 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -0 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ -7 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -0 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -7 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ -3 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ -0 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -8 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ -0 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ -0 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -1 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ -6 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -6 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline \end{array}$$



SEPARATING A SET INTO MORE THAN TWO SUBSETS

Until now, we have separated a set into only two subsets, like this.



Make this equation true.

$$2 + \square = \square$$

Can we separate the set of three cups into more than two subsets?



Does separating the subset like this change the cardinal number of the set?

No, it is still a set of three cups.

How many members are in each subset?

$$\square + \square + \square = 3$$

Separate the set below into three subsets.



Make this equation true.

$$\triangle + \square + \square = 4$$

NOTE: The set of four members can be separated into subsets of:

1 and 1 and 2 or

2 and 1 and 1 or

1 and 2 and 1.

If the child decides to use the empty set, he can also write $3 + 1 + 0$, or $2 + 2 + 0$, or any other arrangement of the above numerals.

JOINING MORE THAN TWO SETS

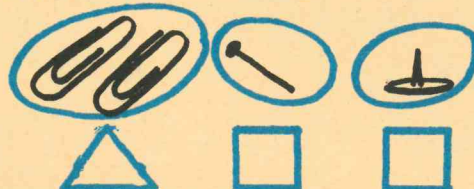
Until now, we have joined only two sets to get a new set.



Finish the equation for this.

$$2 + \square = \square$$

Can we join three sets?



Write the numeral for each set in the number frames.

Can we add these numbers to get a sum?

$$2 + 1 + 1$$

We can join the set of clips to the set of pins.



Then we can join the new set to the set of tacks.

We can add

$$\begin{array}{r} 2+1 \\ 3 \end{array} + \begin{array}{r} 1 \\ 1 \end{array} = \square$$

We can do it another way.

We can add the set of pins to the set of tacks.



Then we can join the set of clips to the new set.

We can add

$$\begin{array}{r} 2 \\ 2 \end{array} + \begin{array}{r} 1+1 \\ 2 \end{array} = \square$$

If you have not already written the sums for the equations, do it now.

The sum each time is the same.

NOTE: This page introduces the associative or grouping property of arithmetic. This property is stated $(a+b) + c = a + (b+c)$.

In other words, it does not matter whether the first 2 addends are added first, or the last 2 addends are added first.

Joining sets and adding numerals are both binary operations; that is, the operation can only be performed on two units at a time.

GROUPING IN ADDITION

The sets below are separated into three subsets.
Write the numerals in the number frames below each subset.

$\square + \triangle + \square$	$\nabla + \nabla + \square$	$\triangle + \square + \square$

Before we can make number sentences showing the sets above,
2 of the subsets will have to be joined.

--	--	--

Make these equations true.

$$\begin{array}{lll} \textcircled{1+3} + 1 = & \textcircled{2+2} + 1 = & 3 + \textcircled{1+1} = \\ \square + 1 = \diamond & \square + 1 = \diamond & 3 + \nabla = \diamond \end{array}$$

Can the subsets be joined in any other order?

--	--	--

Make these equations true.

$$\begin{array}{lll} 1 + \textcircled{3+1} = & 2 + \textcircled{2+1} = & \textcircled{3+1} + 1 = \\ 1 + \square = \diamond & 2 + \triangle = \diamond & \square + 1 = \diamond \end{array}$$

Look at the way the subsets were grouped first.

Look at the way the subsets were grouped next.

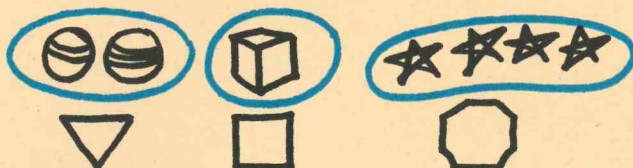
The sums both times were the same.

NOTE: Have the child put together, or draw, other sets up to 5. Then have him separate the sets into 3 subsets. He should realize that only 2 numerals can be added together at a time. This calls for grouping, so that eventually he will only have to add 2 numerals to get the sum. The order in which the subsets are grouped makes no difference.

THE GROUPING PROPERTY OF ADDITION

Can we join three sets?

Write the numeral for each set in the number frame.



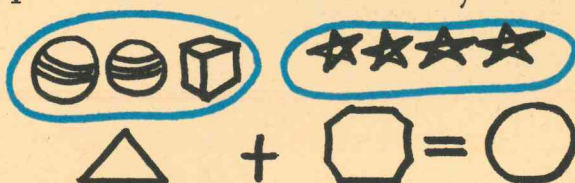
In addition, we can only add two numbers at a time.

We can join the first two sets. Write the equation which shows the number for the new set.

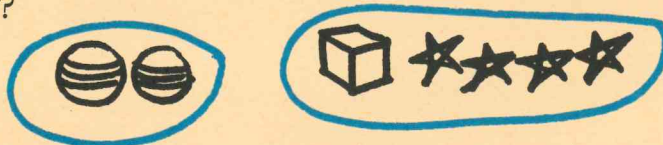
$$\nabla + \square = \triangle$$

Now add the other set to the set you just made.

Write the equation that shows what you did.



When we have three sets to be joined, or three numbers to be added, it takes two operations. Can we join the sets another way?



We can write the equations for this.

$$\nabla + \diamond = \bigcirc$$

The sums in both equations are the same.

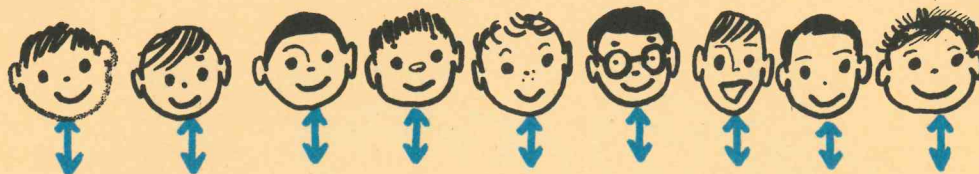
It doesn't matter which groups you joined first, or which numerals you added first.

This is called the *grouping property* of addition.

NOTE: The child should understand that the sum in each equation is the same, whether the first two addends are grouped first, or the last two addends are grouped first.

TEN

Match the set of counting numbers with the set of boys.



The cardinal number of the set of boys is nine.

Draw one more boy next to the set of boys.

There are *ten* boys now.

But there are no symbols in our counting system after 9.

There is one symbol that comes before 1.

The symbol is 0.

Zero is not a counting number, because it does not count anything. We never start to count at zero. The first numeral used in counting is 1.

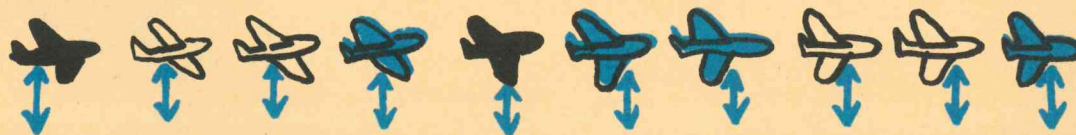
But we can make the numeral for ten by using 1 and 0 together.

The symbol for ten is 10.

Every numeral before 10 was a single symbol.

Ten is made of a combination of two symbols we already know.

Match the set of airplanes below with the set of counting numbers.



NOTE: Ten is a very important number in our counting system. The concept of ten as "nine and one more" must be given first.

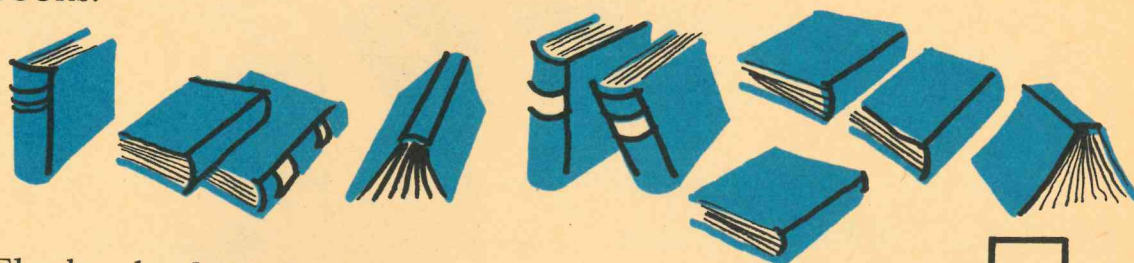
Have the child point out, or collect, or draw, other sets of ten: 10 fingers, 10 pennies, etc., and then write the cardinal number of each set.

He should understand that the only symbols used in our notation systems are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0, but that zero is not a counting number.

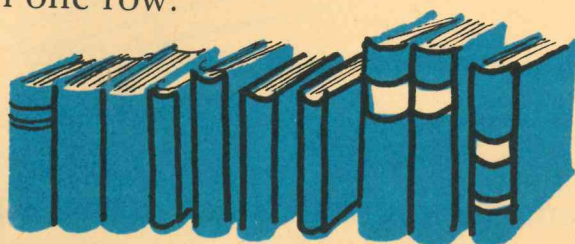
Ten is unusual because it introduces the idea that a numeral can be made of a combination of symbols.

SETS OF TEN

In the number frame below, write the numeral for the set of books.



The books don't look very neat that way, do they?
Let's pretend we are taking the set of 10 books, and putting them in one row.



The books look much better now.

They are easier to count, too.

Instead of having ten books spread out over the page, we now have a neat set of 10 books.

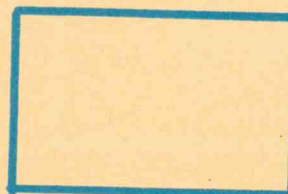
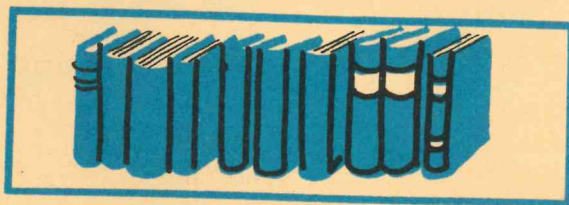
How many sets of ten books are there?

After the books have been put in one set of ten, how many books are left?

We now have 1 set of ten books, and an empty set, or a set of 0 books.

That is why we write ten like this: 10.

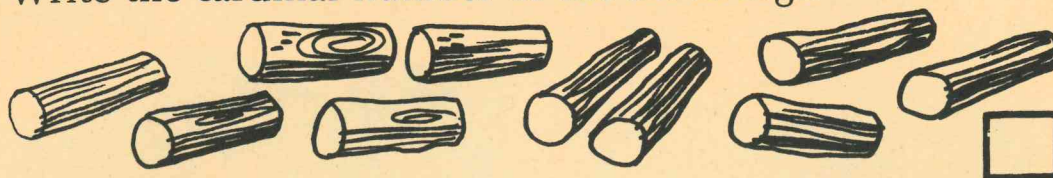
10 means that we have 1 set of ten books, and another set of 0 books.



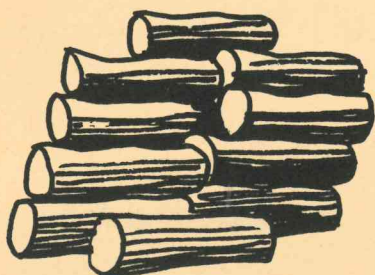
NOTE: This page introduces the idea of grouping by tens, which is the beginning of the concept of place value.

GROUPING BY TEN

Write the cardinal number of the set of logs.



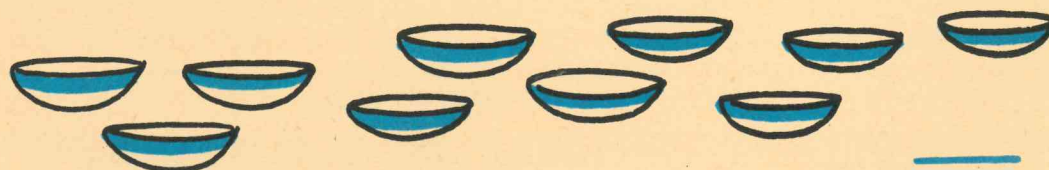
Now let's pretend we can put the logs together into 1 pile. They will look like this.



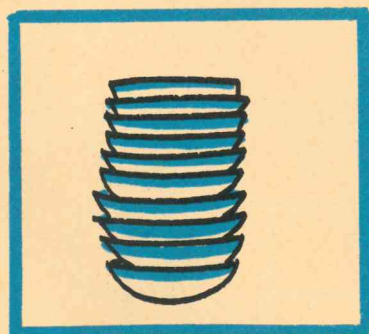
How many sets of ten are there? _____

How many logs are left over? _____

In our counting system you will find that we always group by tens. If we have ten or more elements in a set, we make it easier to count by putting the things into 1 group of ten. Write the numeral for the set of dishes.



The dishes in the frame below are piled into 1 set of ten.



How many sets of ten are there? _____

How many dishes are left over? _____



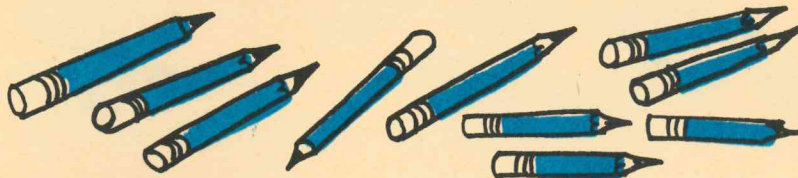
NOTE: This page strengthens the idea of grouping by ten.

TEN AND ONE

TEN AND TWO

TEN AND THREE

Pretend that you can only count to ten.
How could you count the set of pencils below?



You can count the set of 10 pencils.
You could call all the pencils one set of ten, and 1 more.
This number name is *eleven*.
The numeral for eleven is written 11.

How could you count the set of clips below?



You can count the set of ten clips.
You could call all the clips one set of ten, and 2 more.
This number name is *twelve*.
The numeral for twelve is written 12. _____
How could you count the set of checkers below?



You can count the set of ten checkers.
You could call all the checkers one set of ten, and 3 more.
This number name is *thirteen*.
The numeral thirteen is written 13. _____

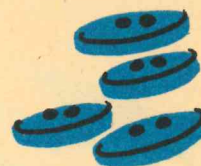
NOTE: The child should be beginning to understand that these two-place numerals actually show 2 sets; a set of ten, and a set of units or ones.

TEN AND FOUR

TEN AND FIVE

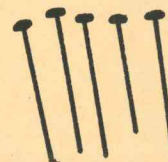
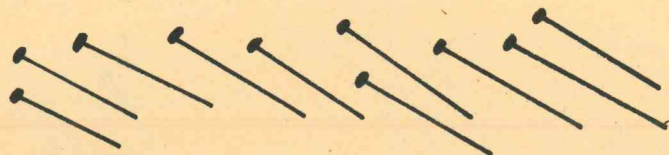
TEN AND SIX

Pretend that you can only count to ten.
How could you count the buttons below?



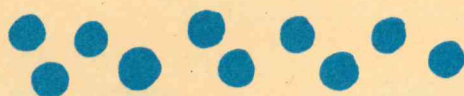
You can count the set of ten buttons.
You could call all the buttons one set of ten, and 4 more.
The number name for this is *fourteen*.
The numeral for fourteen is 14. _____

How could you count the pins below?



You can count the set of ten pins.
You could call all the pins one set of ten, and 5 more.
The number name for this is *fifteen*.
The numeral for fifteen is written 15. _____

How could you count the dots below?



You can count the set of ten dots.
You could call all the dots one set of ten, and 6 more.
The number name for this is *sixteen*.
The numeral for sixteen is 16. _____

NOTE: Lead the child to see that the numeral 1 to the left stands for the 1 set of tens.
The numeral to the right stands for the number in the set of ones.
Point out that *fourteen* is almost like *four and ten*
fifteen is almost like *five and ten*
sixteen is almost like *six and ten*.

SEVENTEEN, EIGHTEEN, NINETEEN

Write the set of counting numbers from 1 to 16.

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

How are the numerals in the first column different from the numerals in the second column?

There is a 1 in each numeral in the second column.

The next numbers in the set of counting numbers are:

seventeen 17

eighteen 18

nineteen 19

Seventeen means 1 set of ten, and _____ more.

Eighteen means 1 set of ten, and _____ more.

Nineteen means 1 set of ten, and _____ more.

Write the set of counting numbers from 1 to 19.

_____	_____	_____	_____	_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____	_____	_____	_____	_____

NOTE: The numerals in the second column above are just like those in the first column, except that each numeral begins with 1.

The 1 means 1 set of ten. The numeral to the left is in the *tens* place.

The child should now be able to read and write numerals to nineteen, and to count sets with up to 19 members.

He might want to count 19 pennies, or 19 cards, or 19 sheets of paper.

Have him group a set into 1 set of ten and the ones left over.

TEN AS AN ADDEND

We said that a set of ten, and 1 more, is written 11.

We can write this

$$10 + 1 = 11$$

We can also write it in the vertical form.

$$\begin{array}{r} 10 \\ + 1 \\ \hline 11 \end{array}$$

$$10 + 2 = \boxed{} + \begin{array}{r} 10 \\ 2 \end{array}$$

$$10 + 3 = \boxed{} + \begin{array}{r} 10 \\ 3 \end{array}$$

$$10 + 4 = \boxed{} + \begin{array}{r} 10 \\ 4 \end{array}$$

$$10 + 5 = \boxed{} + \begin{array}{r} 10 \\ 5 \end{array}$$

$$10 + 6 = \boxed{} + \begin{array}{r} 10 \\ 6 \end{array}$$

$$10 + 7 = \boxed{} + \begin{array}{r} 10 \\ 7 \end{array}$$

$$10 + 8 = \boxed{} + \begin{array}{r} 10 \\ 8 \end{array}$$

$$10 + 9 = \boxed{} + \begin{array}{r} 10 \\ 9 \end{array}$$

NOTE: Lead the child to see that in each example of vertical notation, zero is an addend. Recall that using zero does not change the sum when it is added to another addend. In other words:

0	10	0	10
$\begin{array}{r} +1 \\ \hline 1 \end{array}$	$\begin{array}{r} +1 \\ \hline 11 \end{array}$	$\begin{array}{r} +2 \\ \hline 2 \end{array}$	$\begin{array}{r} +2 \\ \hline 12 \end{array}$

TWENTY

We have said that in our counting system, we group in sets of ten.

In the numeral 10, the 1 means how many sets of ten?

The zero means how many left over?

16 means how many sets of 10?

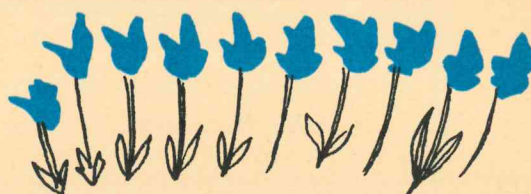
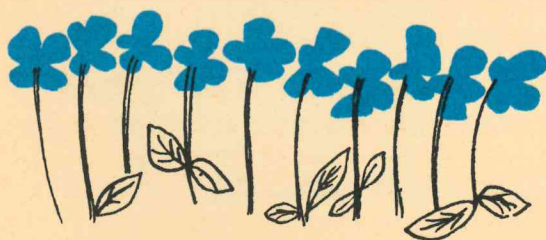
How many left over?

19 means how many sets of 10?

How many left over?

Pretend again that you can only count to ten.

How could you count the set of flowers below?



You can count the set of ten flowers.

You could call all the flowers one set of ten, and ten more.

But that makes 2 sets of ten, doesn't it?

The number name for that number is *twenty*.

The numeral for twenty is written 20.

Can you tell what the 2 means?

What does the 0 mean?

NOTE: Lead the child to see that the 2 in the numeral 20 tells how many sets of ten. There are 2 sets of 10 and 0 flowers left over.

Have the child count out other sets of 20 things, then group them into 2 sets of 10 each. He should associate the word "twenty" and the symbol 20 with 2 sets of 10.

TWENTY-ONE

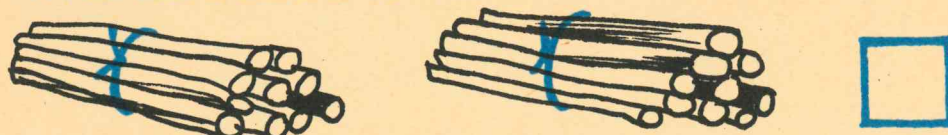
TWENTY-TWO

We have said that the number name for two sets of 10 is twenty.

The numeral for twenty is written 20.

Each pile of sticks that are tied into a bundle below has ten sticks in it.

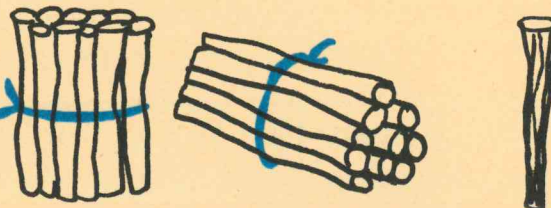
Write the numeral for the number of sticks in all.



Pretend that you can count only to ten.

How many sticks are there below? Each bundle has a set of ten sticks.

You can see that there are
2 sets of ten and
1 more stick.



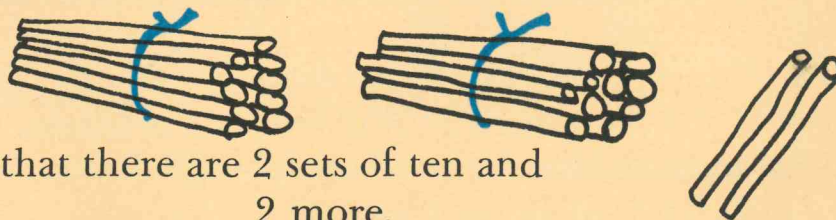
You know that the numeral for 2 sets of 10 is 20.

The number name for two sets of ten, and 1 more, is *twenty-one*.

The numeral for twenty-one is written 21. _____

21 means 2 sets of ten and
1 more.

How could you count these sticks? Each bundle has a set of ten sticks.



You can see that there are 2 sets of ten and
2 more.

The number name for this set of sticks is *twenty-two*.

The numeral for twenty-two is written 22. _____

NOTE: The child should understand that the 2 on the left of a two-digit numeral tells the number of sets of 10. The 2 on the right tells the number of single sticks, or units.

NUMERALS TO 39

We have said that in our counting system we group in sets of ten.

In a numeral with two symbols, the symbol on the left tells the number of sets of ten.

23 means 2 sets of 10 and 3 more



The symbol on the right tells the number of things after the set has been grouped by tens.

23 means _____ tens and _____ single things.

24 means _____ tens and _____ more.

25 means _____ tens and _____ more.

Write the numeral to name each number below.

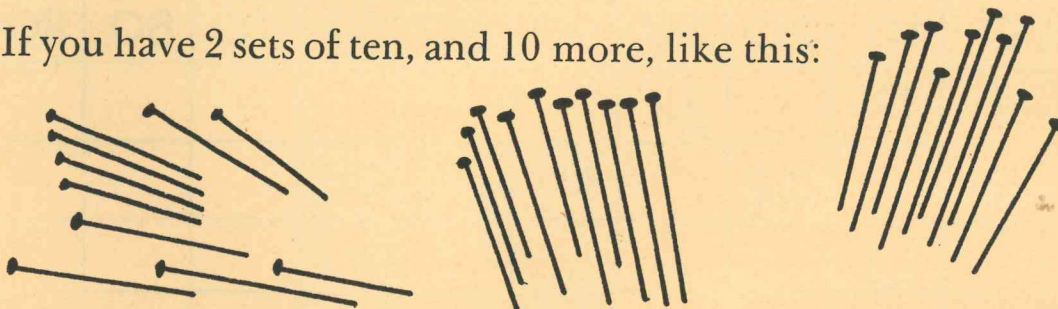
2 sets of ten, and 6 more _____

2 sets of ten, and 7 more _____

2 sets of ten, and 8 more _____

2 sets of ten, and 9 more _____

If you have 2 sets of ten, and 10 more, like this:



the 10 things can be grouped into another set of ten.

Three sets of ten are called *thirty*.

The numeral for thirty is written 30.

Write the numeral to name each number below:

3 sets of ten, and 1 more _____

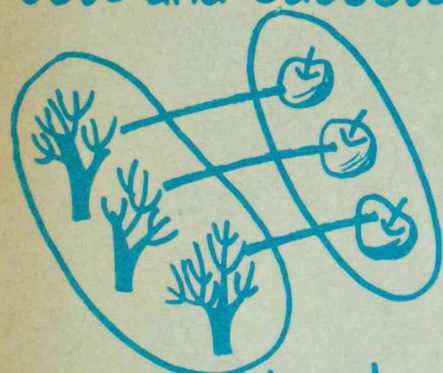
3 sets of ten, and 2 more _____

3 sets of ten, and 9 more _____

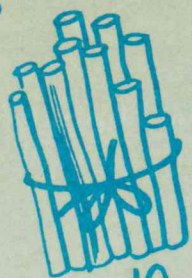


sets and subsets

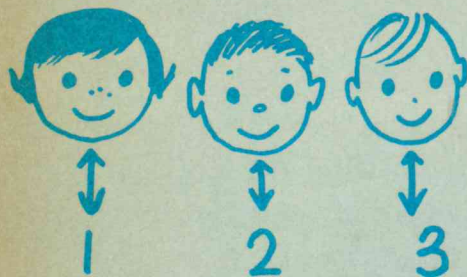
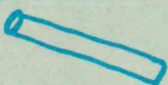
equalities
 $5 + \square = 7$
 $5 < 6$
inequalities



equivalent sets



10
and 1 more

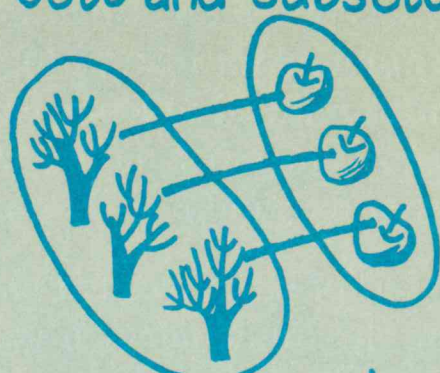


one-to-one
matching

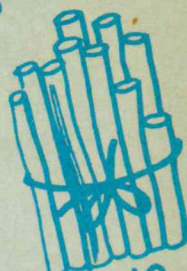


sets and subsets

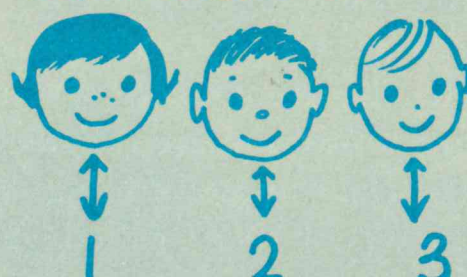
equalities
 $5 + \square = 7$
 $5 < 6$
inequalities



equivalent sets



10
and 1 more



one-to-one
matching

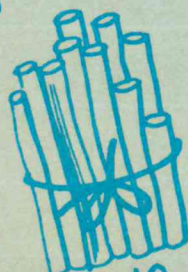


sets and subsets

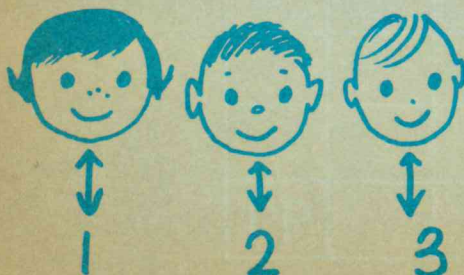
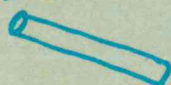
equalities
 $5 + \square = 7$
 $5 < 6$
inequalities



equivalent sets



10
and 1 more



one-to-one
matching

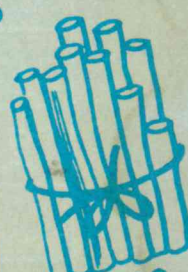


sets and subsets

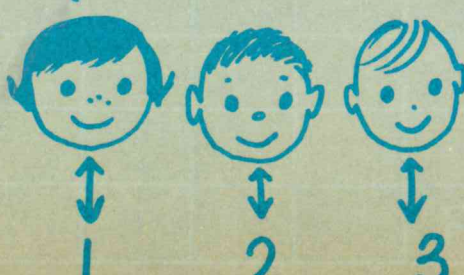
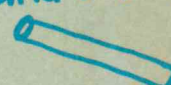
equalities
 $5 + \square = 7$
 $5 < 6$
inequalities



equivalent sets



10
and 1 more



one-to-one
matching

NEW MATHEMATICS PRACTICE WORKBOOK

The teaching of mathematics has recently undergone great changes. Simple arithmetic is now taught so that even at the elementary level, it can lead directly to algebra and logic.

This book will help children, parents, and teachers who want an introduction to New Mathematics or who need additional practice materials in set theory, addition, and subtraction facts to 9, and reading and writing numerals to 99.

Notes on each page further explain the concepts and terminology used for parents and teachers.

You will find answers to these and many other questions:

What is a set?

How are sets joined?

What is an inequality?

Why is subtraction called the inverse of addition?

What is an empty set?

How are the placeholders used in vertical notation?

What is the identity property for addition?

TREASURE BOOKS

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